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On the History of Indian Mathematics

SATYAANSHU
Student, Computer Science & Engineering
R.V.College of Engineering, Mysore Road,
Bangalore-560 059, India

DR.N. SHIVAKUMAR
Professor & Head, Department of Mathematics
R.V.College of Engineering, Mysore Road,
Bangalore-560 059, India

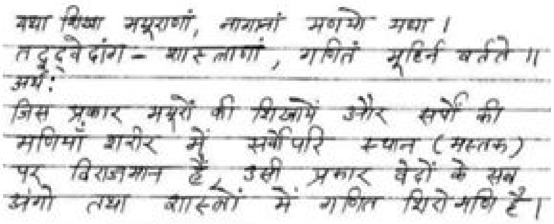
Abstract: **Indian mathematics has its roots in Vedic literature. Between 1000 B.C. and 1800 A.D. various treatises on mathematics were authored by Indian mathematicians in which were set forth for the first time, the concept of zero, numeral system, techniques of algebra and algorithm, square root and cube root. However, despite widely available, reliable information, there is a distinct and inequitable neglect off the contributions of the sub-continent. Many of the developments of Indian mathematics remain almost completely ignored, or worse, attributed to scholars of other nationalities, often European. However a few historians (mainly European) are reluctant to acknowledge the contributions of Indian mathematicians. They believe Indians borrowed the knowledge of mathematics from Greeks. It is a fact that Indians were way ahead than the Greeks and put forth beautiful concepts and theorems which were later borrowed by the Greeks. Mathematics today, owes a huge debt to the outstanding contributions made by Indian mathematicians over many hundreds of years. Such a wonderful history of Indian mathematics cannot be masked by a few Eurocentric historians.**

"India was the motherland of our race
And Sanskrit the mother of Europe's languages
India was the mother of our philosophy
Of much of our mathematics, of the ideas embodied in
Christianity of self-government and democracy
In many ways, mother India is the mother of us all"

\- Will Durant, American historian (1885-1981)

This basic chronology of the history of mathematics was very simple, it had primarily been the invention of the ancient Greeks, whose work had continued up to the middle of the first millennium A.D. Following which there was a period of almost 1000 years where no work of significance was carried out until the European renaissance, which coincided with the 'reawakening' of learning and culture in Europe following the so called dark ages.

Some historians made some concessions, by acknowledging the works of Egyptian, Babylonian,

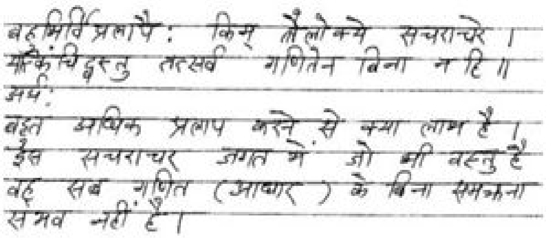

यथा शिखा मयूराणां, नागानां मणयो यथा ।
तद्द्वेदांग-शास्त्राणां, गणितं मूर्धनि वर्तते ॥
अर्थ:
जिस प्रकार मयूरों की शिखायें और सर्पों की
मणियां शरीर में सर्वोपरि स्थान (मस्तक)
पर विराजमान है, उसी प्रकार वेदों के सब
अंगो तथा शास्त्रों में गणित शिरोमणि है ।

Meaning: Just as branches of a peacock and jewel stone of a snake are placed at the highest place of body (Forehead), similarly position of mathematics is highest in all branches of Vedas and Shastras.

Famous Jain mathematician Mahaviracharya has said the following:

Indian and Arabic mathematicians. Indian scholars, on the relatively rare occasions they were discussed, were merely considered to be custodians of ancient Greek learning. Mathematical developments of the Indian sub-continent is not only neglected in histories of mathematics, but also has produced some of the most remarkable results of mathematics. These results, beyond being simply remarkable because of the time in which they were derived, show that several 'key' mathematical tops, andsubsequent results, indubitably originate from the Indian sub-continent.

The book Vedang Jyotish (written 1000 B.C.) mentions the importance of mathematics as follows:


बहुभिर्विप्रलापैः किम् त्रैलोक्ये सचराचरे ।
यत्किंचिद्वस्तु तत्सर्वं गणितेन बिना न हि ॥
अर्थ:
बहुत अधिक प्रलाप करने से क्या लाभ है ।
इस सचराचर जगत में जो भी वस्तु है
वह सब गणित (आधार) के बिना समझना
सम्भव नहीं है ।

Meaning: What is the use of much speaking, whatever object exists in this moving and non-moving world, cannot be understood without the base of mathematics.

Indian scholars made vast contributions to the field of mathematical astronomy and as a result, contributed mightily to the developments of arithmetic, algebra, trigonometry and secondarily geometry and combinatorics. Perhaps most remarkable were developments in the fields of infinite series expansions of trigonometric expressions and differential calculus. Surpassing all these achievements however was the development of decimal numeration and his place value system, which without doubt stand together as the most remarkable developments in the history of mathematics, and possibly one the foremost developments in the history of human kind. The decimal place value system allowed the subject of mathematics to be developed in ways that simply would not be possible otherwise. It also allowed numbers to be used more extensively and by vastly more people than ever before.

Laplace said on the beautiful number system invented by the Indians:

"The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place vale and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and invention is more readily appreciated when one considers that it was beyond the two greatest men of antiquity, Archimedes and Apollonius".

The chronology of the history of mathematics is not entirely linear. It is complicated than the diagrammatic representations where the work of one group of people (or country) is proceeded by the work of another group and so on.

G.Joseph states:

"…A variety of mathematical activity and exchange between a number of cultural areas went on while Europe was in a deep slumber".

Some of the prominent mathematicians and astronomers who made important contributions to Indian mathematics are:

Apasthamba, Aryabhata I, Aryabhata II, Varahamihira, Baudhayana, Mahavira,Jagannatha, Samrat, Sripati, Yajnavalka,Virasena, Srinivasa Ramanujam,Pingala, Panini, Brahmagupta, Bhaskara I, Bhaskara II.

The history of Indian mathematics can be divided into five parts as:

1) Ancient Period
    a) Vedic Period (around 3000 B.C. – 1000 B.C.)
    b) Post Vedic Period (1000 B.C. – 500 B.C.)
2) Pre Middle period (500 B.C. – 400 A.D.)
3) Middle period or classic period (400 A.D. – 1200 A.D.)
4) End of classic period (1200 A.D. – 1800 A.D.)
5) Current period (After 1800 A.D.)

1. <u>Ancient Period</u>

a) <u>VEDIC PERIOD (3000 B.C. – 1000 B.C.)</u>

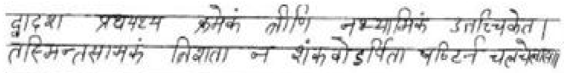
The Vedic religion was followed by the Indo-Aryan people, who originated from the north of the sub-continent. It is through the works of Vedic religion that we gain the literary evidence of Indian culture and mathematics. Written in Vedic Sanskrit, the Vedic works, Vedas and Vedangas are primarily religious in content, but embody a large amount of astronomical knowledge and hence a significant knowledge of mathematics.

The works of this period which contained mathematics are:

- Taittareya Samhita
- Shatapata Brahmana
- Yajurveda
- Atharvaveda

Rig-Veda plus additional Samhitas

Numerals and decimals are clearly mentioned in the Vedas. There is a **Richa** in Veda, which says the following:



In the above **Richa**, Dwadash (12), Treeni (3), Trishat (300) – numerals have been used. This indicates the use of writing numerals based on 10. In this age the discovery of 'Zero' and the '10 place value method' is a great contribution to the world by India in the arena of mathematics.

In the second section of earlier portion of Narad Vishnu Puran, written by Ved Vyas, mathematics is described in the context of Triskandh Jyotish. In that, numbers have been described which are ten times of each other, in a sequence of '10 to the power n' i.e.$\mathbf{10^n}$

Many rules and developments of geometry are found in Vedic works such as:

- Use of geometric shapes, including triangles, rectangles, squares, trapezium and circles.
- Equivalence through numbers and area
- Equivalence led to the problem of squaring the circle and vice versa
- Early problems of Pythagoras theorem

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- Estimation for π – Three values for π are found in Shatapata Brahmana.

It seems most probable that they arose from transformations of squares into circles and circles to squares. The values are:

$_1 = {}^{25}/_{8}$ (3.125)
$_2 = {}^{900}/_{289}$ (3.11418685...)
$_3 = {}^{1156}/_{361}$ (3.202216...)

All the four arithmetical operators (addition, subtraction, multiplication and division) are found in Vedic works. This proves, at that time various mathematical methods were not in concept stage, rather those were getting used in a methodical and expanded manner.

b) <u>POST VEDIC PERIOD (1000 B.C. – 500 B.C.)</u>

To perform rituals, altars were to be constructed. If this ritual sacrifice wasto be successful, then the altar had to conform to very precise measurements. To make those precise measurements, geometrical mathematics was developed. The rules were available in the form of Shulv sutras (also Sulbasutras). Shulv means rope. This rope was used in measuring geometry while making altars.

<u>SULBASUTRAS</u>

The sulbasutras do not contain any proofs of the rules which they describe. The most important sulbasutras are the Baudhayana sulbasutra written about 800 B.C., the Manava sulbasutra written about 750 B.C. and the Katyayana sulbasutra written about 200 B.C.

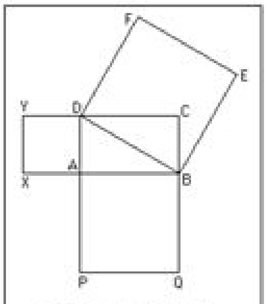
The Baudhayana sulbasutra gave a special case of Pythagoras theorem explicitly:

"The rope which is stretched across the diagonal of a square produces an area double the size of the original square".

The Katyayana sulbasutra gives a more general version:

"The rope which is stretched along the length of the diagonal of a rectangle produces an area which the vertical and the horizontal sides make together".

The diagram illustrates the result:



The Katyayana form of Pythagoras's theorem

Baudhayana uses different approximations for π when constructing circular shapes. Constructions are given which are equivalent to taking π equal to 676/225 (where 676/225 = 3.004), 900/289 (where 900/289 = 3.114) and to 1156/361 (where 1156/361 = 3.202)

<u>PANINI</u>

Panini's work 'Ashtadhyay' provided an example of a scientific notational model that could have propelled later mathematicians to use abstract notations in characterizingalgebraic equations presenting algebraic theorems in a scientific format.

<u>JAIN MATHEMATICS</u>

Jain Acharyas contributed extensively to the development of mathematics. There are elaborated descriptions of mathematics in Jain literatures. Surya Pragyapti and Chandra Pragyapti are two famous scriptures of Jaina branch of ancient India. These describe the use of mathematics.

The main topics of Jaina mathematics were:

- The theory of numbers
- Arithmetical operators
- Geometry
- Operations with fractions
- Simple equations
- Cubic and Quadratic equations
- Permutation and Combination

Ellipse is cleverly described in Surya Pragyapti. Jainas developed a theory of the infinite containing five levels of infinity: Infinite in one direction, in two directions, in area, infinite everywhere and perpetually infinite. They had a primitive understanding of indices and some notion of logarithms to base 2.

Buddhist literature also gave due importance to mathematics. They divided mathematics under two categories:

- Garna (Simple mathematics)

- Sankhyan (Higher mathematics)

They have described numbers under three categories:

- Sankheya (Countable)
- Asankheya (Uncountable)
- Anant (Infinite)

BAKSHALI MANUSCRIPT

The Bakshali manuscript, which was unearthed in the 19th Century, does not appear to belong to any specific period. A few historians class it as a work of the early classical period and others suggest it may be a work of Jaina mathematics. However there is still a debate surrounding the date of the Bakshali manuscript.

The Bakshali manuscript was written on leaves of Birch in Sarada characters and in the Gatha dialect, which is a combination of Sanskrit and Prakrit. The Bakshali manuscript highlights developments in arithmetic and algebra. The arithmetic contained within the work is of high quality. There are eight principal topics in the Bakshali manuscript:

- Examples of the rules of three(and profit and loss and interest)
- Solution to linear equations with as many as five unknowns
- Solutions of quadratic equations
- Arithmetic and Geometric Progressions
- Compound Series
- Quadratic Indeterminate equations (origin of type $y = \frac{ax}{c}$)
- Simultaneous Equations

Advances in notation including use of zero and negative sign took place. There was an improved method for calculating square root, which allowed extremely accurate approximations to be calculated:

$$\sqrt{A} = \sqrt{a^2 + r} = a + \frac{r}{2a} - \frac{(\frac{r}{2a})^2}{2(a + \frac{r}{2a})}$$

The reason for the composition of the Bakshali manuscript is unknown but it seems possible that the motivation was to bring out the developments of mathematics during the time period.

2. PRE MIDDLE PERIOD (500 B.C. – 400 A.D.)

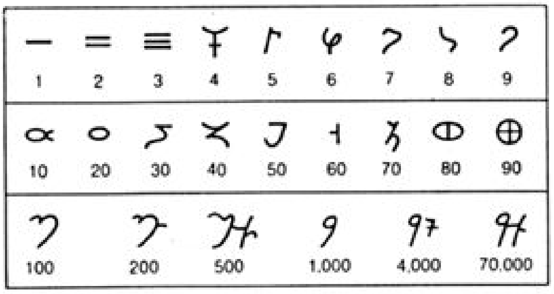
Almost all the writings of this time are lost, except for a few books and few pages of Vaychali Ganit, Surya Siddhanta and Ganita Anuyog. During this period too, mathematics underwent sufficient development. Sathnananga Sutra, Bhagavathi Sutra and Anuyogdwar Sutra are famous books of this time. Apart from these, the books titled Tatvarthaadigyam Sutra Bhashya of Jain philosopher Omaswati (135 B.C.) and Tiloyapannati of Acharya Yativrisham (176B.C.) are famous writings of this time.

Vaychali Ganit discusses in detail the following:

- Basic calculations of mathematics
- Numbers based on 10
- Fractions
- Squares and Cubes of numbers
- Rule of false position
- Interest methods

Sathananga Sutra has mentioned five types of infinite and Anuyogdwar Sutra has mentioned four types of measure and also describes permutation and combination and some rule of exponents. Roots of modern trigonometry lie in the book Surya Siddhanta.

By around 3rd century B.C., Brahmi numerals began to appear. Here is one style of Brahmi numerals.



BRAHMI NUMERALS

3. THE CLASSICAL PERIOD OR MIDDLE PERIOD (400 A.D. – 1200 A.D.)

Indian Mathematics suffered a kind of slump following the Bakshali manuscript. This was probably due to the massive communication problems, but also undoubtedly to the huge political upheaval that took place between the 2nd and 4th centuries A.D., prior to the capturing of power of most of India by the imperial Guptas. Following the establishment the ascent of a 'galaxy' of mathematician-astronomers led by Aryabhata. These men were, first and foremost astronomers, but the mathematical requirements of astronomy (and no doubt further interest) led them to developing many areas of mathematics. The vast majority of the works of this period were, however, in effect, Siddhantas.

ARYABHATA

Aryabhata, who is occasionally known as Aryabhata I, or Aryabhata the elder to distinguish him from a 10^{th} Century astronomer of the same name, stands a pioneer of the revival of Indian mathematics, and the so called 'classical period' or 'golden era' of Indian mathematics.

LIFE AND WORK

Aryabhata was born in 476 A.D., as he writes that he was 23 years old when he wrote his most significant work Aryabhatiya in 499 A.D. He was a member of Kusumapura School.

The extant work of Aryabhata is his key work, the 'Aryabhata-siddhanta', but more famously the 'Aryabhatiya', a concise astronomical treatise of 119 verses written in a poetic form, of which 33 verses are concerned with mathematical rules. The rules didn't contain any proofs.

In mathematical verses of Aryabhatiya, the following topics are covered:

Arithmetic:

- Method of Inversion
- Various arithmetical operators, including the cube and cube root are thought to have originated from Aryabhata's work.
- Aryabhata can also reliably be attributed with credit for using relatively 'new' functions of squaring and square rooting.

Algebra:

- Formulas for finding the sum of several types of series.
- Rules for finding the number of terms of an arithmetical progression
- Rule of three – improvement on Bakshali manuscript
- Rules for solving examples on interest – which led to the quadratic equation.

Trigonometry:

- Tables of Sines. Gupta comments: **"The Aryabhatia is the first historical work of the dated type, which uses some of these (trigonometric) functions and contains a table of sines".**
- Spherical trigonometry.

Geometry:

- Area of triangle
- Similar triangles
- Volume rules

We can see the use of 'word numerals' and 'alphabet numerals' in Aryabhata's work. This was not due to the absence of a satisfactory system of numeration but because it was helpful in poetry. The work of Aryabhata also affords a proof that 'the decimal system was well in vogue'.

Of mathematics contained within the Aryabhatia, the most remarkable is an approximation of π.

$\pi = 3.1416$

He wrote that if 4 is added to 100, then multiplied by 8 then added to 62,000 the answer will be equal to circumference of a circle of diameter 20000. Aryabhata was aware that it was an irrational number and that his value was an approximation, which shows his incredible insight.

In field of pure mathematics, his most significant contribution was his solution to indeterminate equation:$\mathbf{ax - by = c}$.

Aryabhata's work on astronomy was also pioneering, and was far less tinged with a mythological flavour. He even computed the circumference of the earth as 25835 miles which is close to modern day calculation of 24900 miles.

The Aryabhatia was translated into Arabic by Abul al-Ahwazi (before 1000 A.D.) as Zij al-Arjabhar and it is partly through this translation that Indian computational and mathematical methods were introduced to the Arabs, which had a significant effect on the forward progress of mathematics. The work of Aryabhata was also extremely influential in India. Aryabhata died in 550 A.D. This remarkable man was a genius and continues to baffle many mathematicians of today.

BHASKARA I

Bhaskara I continued where Aryabhata left off and discussed in detail further topics such as:

- Longitudes of planets
- Conjunctions of the planets with each other and with bright stars
- Rising and setting of the planets
- Lunar crescent

He expanded on the trigonometric equations provided by Aryabhata. Bhaskara I correctly assessed pi to be irrational. His most important contribution was his formula for calculating sine function which was 99% accurate. He also did pioneering work on indeterminate equations and considered for the first time quadrilaterals with all the four sides unequal and none of the opposite sides parallel.

VAHAMIHIRA

A famous mathematician/astronomer during this period was Varahamihira. He is thought to have lived from 505 A.D. till 587 A.D. and made only fairly small contributions to mathematics. However he increased the stature if the Ujjain school while working there, a legacy that was to last for a long period. Although his contributions to mathematics were small, they were of some importance.

They included:

- Several trigonometric formulas
- Improvement of Aryabhata's sine tables
- Derivation of the Pascal triangle by investigating the problem of computing binomial coefficients.

BRAHMAGUPTA

Following Aryabhata's death around 550 A.D., the work of Brahmagupta resulted in Indian mathematics attaining an even greater level of perfection.

LIFE AND WORK

Brahmagupta was born in in 598 A.D. in Bhinmal city in the state of Rajasthan of northwest India. He was the head of the astronomical observatory at Ujjain and during his tenure he wrote four texts on mathematics and astronomy:

- The Cadamelaka in 624 A.D.
- The Brahmasphutasiddhanta in 628 A.D.
- The Khandakadyaka in 665 A.D.
- The Durkeamynarda in 672 A.D.

The Brahmasphutasiddhanta (corrected treatise of Brahma) is arguably his most famous work. The world's mathematical content was of an exceptional quality.

Brahmagupta had a plethora of criticism directed towards the work of rival astronomers, and in his Brahmasphutasiddhant is found one of the earliest attested schisms among Indian mathematicians. The division was primarily about the application of mathematics to the physical world rather than about the mathematics itself. In Brahmagyota's case, the disagreements stemmed largely from the choice of astronomical chapters and theories. Critiques of rival theories appear throughout the first ten astronomical chapters and the eleventh chapter is entirely devoted the criticism of these theories, although no criticism appear in the twelfth till eighteen chapters.

Brahmagupta's Brahmasphutasiddhanta is composed in elliptic verse, as was common practice in Indian mathematics, and subsequently has a poetic ring to it. As no proofs are given, it is not known how Brahmagupta's mathematics was derived, In the Brahmasphutasiddhant among the major developments are those in the areas of:

1) Arithmetic:

Brahmagupta possessed a greater understanding of the number system (and the place value system) than any one to that point. In the beginning of chapter 12, entitled calculation, he details operations on fractions. He explains how to find the cube and cube root of an integer and later gives rules facilitating the computation of squares and square roots.

He then gives rules for dealing 5 types of combination of fractions:

$$\frac{a}{c}+\frac{b}{c}, \quad \frac{a}{c}.\frac{b}{d}, \frac{a}{1}+\frac{b}{d}, \frac{a}{c}+\frac{b}{d}.\frac{a}{c}=\frac{a(b+d)}{cd},$$

$$\frac{a}{c}-\frac{b}{d}.\frac{a}{c}=\frac{a(d-b)}{cd}$$

Brahmagupta then goes on to give the sum of the squares and cubes of the first n integers. He found the result in terms of the sum of the first n integers rather than in terms of n as is the modern practice. Brahmagupta made use of an important concept in mathematics, the number 'zero'. The Brahmasphutasiddhanta is the earliest known text to treat zero as a number in its own right, rather than as simply a placeholder digit in representing another number as was done by the Babylonians or as a symbol for lack of quantity as was done by Ptolemy and the Romans. In chapter 19, he describes operations on negative numbers. He also describes addition, subtraction and multiplication. Brahmagupta was the first to attempt to divide by zero, and while his attempts of showing

$$\frac{n}{0}=\infty$$

were not ultimately successful they demonstrate an advanced understanding of an extremely abstract concept.

2) Algebra:

Brahmagupta gave the solution of the general linear equation ax + c = by in chapter 18. He further gave two equivalent solutions to the general quadratic equation which are respectively:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

He went on to solve systems of simultaneous indeterminate equations stating that the desired variable must first be isolated, and then the equation must be divided by the desired variable's coefficient. The algebra of Brahmagupta is syncopated. In chapter 12, Brahmagupta finds

Pythagorean triplets for a given length m and an arbitrary multiplier x.

Let:

$$a = mx$$
$$b = m + \frac{mx}{x+2}$$

Then: a and b form a Pythagorean triplet.

Brahmagupta went on to give a recurrence relation for generating solutions to certain instances of Diophantine equations of the second degree such as

$$\mathbf{Nx^2 + 1 = y^2}$$

by using the Euclidean algorithm. The Euclidean algorithm was known to him as "Kuttaka" (Pulveriser) since it breaks numbers down into even smaller pieces. The key to his solution was the identity:

$$\mathbf{(x_1^2 - Ny_1^2)(x_2^2 - Ny_2^2) = (x_1x_2+Ny_1y_2)^2 - N(x_1y_2+x_2y_1)^2}$$

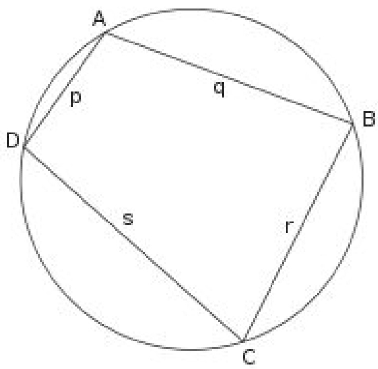
Using this identity and the fact that if $\mathbf{(x_1,y_1)}$ and $\mathbf{(x_2, y_2)}$ are solutions to the equations $\mathbf{x^2\text{-}Ny^2=k_1}$ and $\mathbf{x^2\text{-}Ny^2=k_2}$, respectively, then $\mathbf{(x_1x_2 + Ny_1y_2, x_1y_2 + x_2y_1)}$ is a solution to $x^2\text{-}Ny^2=k_1k_2$. He was able to find integral solutions of the equation $\mathbf{x^2\text{-}Ny^2=k_i}$

3) Geometry:

Brahmagupta's most famous result in geometry is his formula for cyclic quadrilaterals. Given the lengths of the sides of any cyclic quadrilateral, he gave an approximate and an exact formula for the figure's area.

So, given the lengths p, q, r of a cyclic quadrilateral, the approx. area is $(\frac{p+r}{2})(\frac{q+s}{2})$ while letting t = $(\frac{p+q+r+s}{2})$

Exact Area = $\sqrt{(t-p)(t-q)(t-r)(t-s)}$



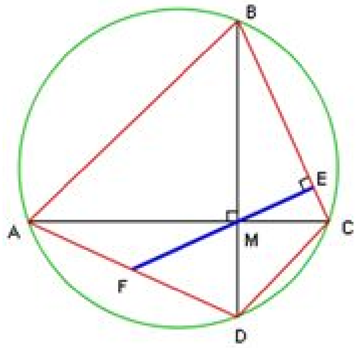
One of his theorems on triangles states that when the base is divided by its altitude, the lengths of two segments are:

$$b \pm \frac{(c^2 - a^2)}{b}$$

He further gives a theorem on rational triangles. A triangle with rational sides a, b, c and rational area is of the form

$a = \frac{1}{2}\left(\frac{u^2}{v} + v\right)$, $b = \frac{1}{2}\left(\frac{u^2}{w} + w\right)$, $c = \frac{1}{2}(\frac{u^2}{v} - v + \frac{u^2}{w} - w)$

For some rational numbers u, v, w.



Brahmagupta's theorem states that **AF = FD**.[10]

He also gives formulas for the lengths and areas of geometric figures, such as the circumradius of an isosceles trapezoid and a scalene quadrilateral, and the lengths of diagonals in a scalene cyclic quadrilateral. In verse 40, he gives value for π.

4) Astronomy:

It was through Brahmasphutasiddhanta that the Arabs learned of Indian astronomy. In chapter 7 of his Brahmasphutasiddhanta, entitled 'Lunar crescent', Brahmagupta rebuts the idea that the moon is father from the earth than the sun, an idea which is maintained in scriptures. He does this by explaining the illumination of the moon by the sun. He explains that since the moon is closer to the earth than the sun, the degree of the illuminated part of the moon depends on the relative positions of the sun and the moon, and this can be computed from the size of the angle between the two bodies. He gave methods for calculating the position of heavenly bodies over time, their rising and setting, conjunctions and the calculation of the solar and lunar eclipses.

About the earth's gravity he said:**"Bodies fall towards the earth as it is in the nature of the earth to attract bodies, just as it is in the nature of the water to flow".**

A translation of Brahmasphutasiddhanta was carried out by Muhammad al-Fazari and had a far reaching influence on subsequent Arabic works. In 860 A.D. an extensive commentary on Brahmasphutasiddhanta was written by Prthudakasvami. His work was extremely elaborate and unlike many Indian works did not 'suffer brevity' of expression. Brahmagupta passed away in 668 A.D.

MAHAVIRACHARYA

Mahaviracharya, a Jain by religion, is the most celebrated mathematician of the 9th century. His major work Ganitasar Sangraha was written around 850 A.D. Mahaviracharya was aware of the Jaina mathematicians and also the works of Aryabhata and Brahmagupta, and refined and improved much of their work. He was a member of the mathematical school at Mysore in the south of India. His major contributions to mathematics include:

1) Arithmetic:

- Detailed operations with fractions(and unit fractions)
- Geometric progressions – He gave almost all the required formulas
- General formula for permutation and combination.

2) Algebra:

- Work on quadratic equations
- Indeterminate equations
- Simultaneous equations

3) Geometry:

- Definitions for most of the geometric shapes
- Repeated Brahmagupta's construction for cyclic quadrilateral
- He referred to the ellipse and gave its perimeter.

Following Mahaviracharya, the most notable mathematician was Prthudakasvami (830 A.D. – 890 A.D.), a prominent Indian algebraist who is best known for his work on solving equations. He wrote a commentary on Brahmagupta's Brahmasphutasiddhanta.

SRIDHARA

In the early 10th century a mathematician by the name of Sridhara (870 A.D. – 930 A.D.) lived. However, it's a fact that he wrote Patganita Sara, a work on arithmetic and mensuration. It contained exactly 300 verses and is hence also known by the name Trisatika. It contains the following topics:

- Rules on extracting square and cube roots
- Operations on functions
- Eight rules for operations involving zero
- Theory on cyclic quadrilaterals with rational sides
- A section concerning rational solutions of various equations of the Pell's type
- Methods for summation of different arithmetic and geometric series

It is thought that Sridhara also composed a text on algebra. The legacy of Sridhara's work was that it had some influence on the work of Bhaskaracharya II.

ARYABHATTA II

A mathematical-astronomer, by name Aryabhata II (920 A.D. – 1000 A.D.) made important contributions to algebra. In his work Mahasiddhanta he gives 20 verses of detailed rules for solving **by = ax + c**

SRIPATI

Sripati (1019 A.D. – 1066 A.D.) was a follower of the teaching of Lalla and in fact the most important Indian mathematician of the 11th century. He impressively gave the identity:

$$\sqrt{(x+\sqrt{y})} = \sqrt{\frac{x+\sqrt{x^2-y}}{2}} + \sqrt{\frac{x-\sqrt{x^2-y}}{2}}$$

BHASKARACHARYA II

Bhaskaracharya or Bhaskara II is regarded almost without question as the greatest Hindu mathematician of all time and his contribution to not just India, but world mathematics.[9]

LIFE AND WORK

He was born in 1114 A.D. in Vijayapura. He belonged to Bijjada Bida. He became the head of the Ujjain school of mathematical astronomy. C.Srinivasiengar claims he wrote Siddhanta Siromani in 1150 A.D., which contained 4 sections:

- Lilavati (arithmetic)
- Bijaganita (algebra)
- Goladhyaya (sphere/celestial globe)
- Grahaganita (mathematics of the planets)

Lilavati is divided into 13 chapters and covers many branches of mathematics, arithmetic, algebra, geometry and a little trigonometry and mensuration. Lilavati is written in poetic form with a prose commentary. Tradition has it that Bhaskara named this work after daughter in order to console her. His astrological meddling coupled with an unfortunate twist of fate is said to have deprived her of her only chance of marriage and happiness.

The contents of Lilavati include:

- Properties of zero (including division)
- Estimation of π
- Methods of multiplication and squaring
- Indeterminate equations

- Integer solutions (first and second order)

Bijaganita is effectively a treatise on algebra and contains the following topics:

- Positive and negative numbers
- Surds
- Simple equations(indeterminate of second, third and fourth degree)
- Indeterminate quadratic equations ($ax^2+b = y^2$)
- Quadratic equations with more than one unknown

Bhaskara derived a cyclic, 'Cakraval' method for solving equations of the form ($\mathbf{ax^2 + bx + c = y}$). He investigated regular polygons up to those having 384 sides, thus obtaining a good approx. value of $\pi = 3.141666$.

Bhaskara is thought to be the first to show that: $\mathbf{d\,(\sin x) = \cos x\, dx}$

Evidence suggests that Bhaskara was fully acquainted with the principle of differential calculus. Bhaskara goes deeper into the differential calculus and suggests that the differential coefficient vanishes at an extremum value of the function, indicating knowledge of concept of 'infinitesimals'. He also gave the well-known results for $\mathbf{\sin(a+b)}$and$\mathbf{\sin(a-b)}$. There is also evidence for early form of Rolle's Theorem:

$\mathbf{If\ f(a) = f(b) = 0, then\ f'(x) = 0\ for\ some\ x\ with\ a} < x < b$[15]

His work Siddhanta Siromani is an astronomical treatise and contains his observations of conjunctions, geography and mathematical techniques etc.

4) END OF CLASSICAL PERIOD OR POST MIDDLE PERIOD (1200 A.D – 1800 A.D.)

The work of Bhaskara was considered the highest point in Indian mathematics attained, and it was long considered that Indian mathematics ceased after that point. Extreme political turmoil through much of the sub-continent and Mongol invasions shattered the atmosphere of discovery and learning and led to the stagnation of mathematical developments as scholars contented themselves with duplicating earlier works.

There were occasional small developments and attempts to revive learning, but nothing of the magnitude of the previous millennium. Worth of a brief mention are Kamalakara (1616 A.D. – 1700 A.D.) and Jagannatha Samrat (1690 A.D. – 1750 A.D.). Both combined tradition ideas of Indian astronomy with Arabic concepts, Kamalakara gave trigonometric results of interest and Samrat made several Sanskrit translations of Arabic versions of Greek works, including notably Euclid's elements. Under the patronage of monarch Sawai Jayasinha Raja, Samrat, along with a group of scholars, attempted to 'reinvigorate' science and learning in India. Though the efforts were not wholly successful, they were in the greatest of faith and should be applauded.

THEKERALA SCHOOL OF MATHEMATICS

Despite the political turmoil, mathematics continued to a high degree in the south of India up to the 16th century. The south of India avoided the worst of the political upheavals of the subcontinent and the Kerala School of mathematics flourished for some time, producing some truly remarkable results.

Madhava (14th century A.D) gave a series expansion of the cos and sine functions. His contributions were instrumental in taking mathematics to the next stage, that of modern classical analysis. Nilakantha (15th century A.D.) extended elaborations on the planetary theory and commentaries on Nilakatha's Tantrasangraha. Chirtabhanu (16th century A.D.) gave integer solutions to twenty types of systems of two algebraic equations, using both algebraic and geometric methods in developing his results. There are seven forms:

$$\mathbf{x + y = a}, \quad \mathbf{x - y = b},$$
$$\mathbf{x^2 + y^2 = d}, \quad \mathbf{x^2 - y^2 = e},$$
$$\mathbf{x^3 + y^3 = f}, \quad \mathbf{x^3 - y^3 = g}$$
$$\mathbf{xy = c}$$

For each case, Chitrabhanu gave an explanation and justification of his rule as well as an example.[3]

Important discovery by Keralese mathematicians include:

- Infinite series
- Expansions of trigonometric functions
- Newton-Gauss interpolation formula
- Sum of an infinite series and

It must be considered most unfortunate that a country, which on reflection was unarguably a world leader in the field of mathematics for several thousands of years ceased to contribute in any significant way.

5) CURRENT PERIOD (1800 A.D. ONWARDS)

Bapudev Shastri (1813 A.D.) wrote books on geometrical mathematics, numerical mathematics and trigonometry. Sudhakar dwivedi (1831 A.D.) wrote books titled:

- Deergha Vritta (dealing with ellipse)
- Golaya Rekha Ganit (sphere line mathematics)
- Samikaran Meemansa (analysis of equations)

SRINIVASA RAMANUJAN

Srinivasa Ramanujan (1889 A.D.) is a modern mathematics scholar. He followed Vedic style of writing mathematical concepts in terms of formulae and then proving it. His intellectuality is proved by the fact; it took all mettle of current mathematicians to prove a few out of his 50 theorems. Ramanujan also showed that any big number can be written as sum of not more than four prime numbers.

He further showed how to divide the number into two or more squares or cube. He also stated that 1729 is the smallest number which can be written in the form of sum of cubes of two numbers in two ways, i.e. $1729 = 9^3 + 10^3 = 1^3 + 12^3$. Since then the number 1729 is called Ramanujan's Number.

Swami Bharti Krisnateerthaji (1884 A.D. – 1960 A.D.)

Swami Bharti Krisnateerthaji wrote the book Vedic Ganit. He is the **founder** and **father of Vedic Ganit**.Bharati Krishnaji got the key to Ganita Sutra coded in the Atharva Veda and rediscovered Vedic Mathematics with the help of **lexicographs**. He found **"Sixteen Sutras"** or word formulas which cover **all the branches of Mathematics** - Arithmetic, Algebra, Geometry, Trigonometry, Physics, plan and spherical geometry, conics, calculus- both differential and integral, applied mathematics of all various kinds, dynamics, hydrostatics and all.[12]

SHAKUNTALA DEVI (1939A.D. – 2013 A.D.)

Sakuntala Devi has written many books on mathematics. She was also the personal astrologer of the President of India.Her feats include:In 1980, she gave the product of two, thirteen digit numbers within 28 seconds, many countries have invited her to demonstrate her extraordinary talent.In Dallas she competed with a computer to see who give the cube root of 188138517 faster, she won! At university of USA she was asked to give the 23rd root of a 201 digit number. She answered in 50 seconds. It took a UNIVAC 1108 computer, full one minute to confirm that she was right after it was fed with 13000 instructions

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